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Spin-Dependent Forces and Current Quark Masses

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The $J = (3/2)\Delta$, $J = 1/2$ Nucleon mass difference shows the quark energies can be spin dependent. It is natural to expect that the quark wave functions also depend on spin. A spin-dependent quark force is fitted to the proton and neutron magnetic moments, axial charge, and spin content using a $(1/2^+)^3$ configuration for the quarks and assuming only zero mass u and d quarks are in the nucleon. In the octet, such spin-dependent forces lead to different wave functions for quarks with spin parallel or antiparallel to the nucleon spin. The eigen-energy of this potential is 0.15 GeV higher for quark spin parallel than for the quark spin antiparallel to the proton spin. This potential predicts a single quark energy of 0.37 GeV for mass-less quarks in the Delta. Assuming the quark forces are flavor independent, this potential predicts magnetic moments of a bound strange quark to be very close to those determined empirically from the octet magnetic moments.

KEY WORDS: current mass quarks; spin-dependent potentials; octet magnetic moments; proton spin content.

1. INTRODUCTION

The three-quark model with up, down, and strange quarks can explain most of the properties of the nucleon, the spin 1/2 octet ground state, and the spin $(3/2)\Delta$ decuplet state. The three-quark model predicts the neutron proton magnetic moment ratio to be $-2/3$, in close agreement with the experimental (Hagiwara *et al.*, 2002) ratio of -0.684979. Lipkin (1971, 1990) also considered the nucleon axial charge ratio and the spin carried by the quarks as well as the magnetic moments. He showed that the quarks carried only about 70% of the nucleon spin when the axial charge ratio |Ga/Gv| is reproduced. Baryon magnetic moments have been mostly analyzed (Silvestre-Brac, 1997) under six main hypotheses.

(1) The baryons are pure three quarks systems and meson exchange currents are negligible. Mesons may have given rise to the potential used, but they are left out of the electromagnetic current.

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(2) A nonrelativistic potential model, based on the Schrodinger or Salpeter eq- uations is used. The spin degrees of freedom are treated a la Pauli, and not a la Dirac.
(3) There is no anomalous magnetic moment for the quarks. The gyro magnetic ratio for a quark is exactly 2, and there is no quark form factor.
(4) There are no tensor forces between the quarks. Tensor forces because of gluon and pion exchanges do exist, but in practice their effect on the wave function configuration is very weak.
(5) The u and d quarks have the same mass, they are considered as two components of an isospin doublet.
(6) The space wave function is not really needed; it has to be completely symmetric under permutations of identical particles. Very often a trial function of Gaussian type is employed.

Hypothesis (6) is often given up, and sometimes replaced with a less restrictive hypothesis (6a) that there be no totally antisymmetric component to the wave function. But even with (6a), the joint application of hypotheses (1–5) is inconsistent with the experimental magnetic moments. But [4] even giving up the isospin symmetry of (5), the set of hypotheses (1–4) are inconsistent with the experimental data. The conclusion of [4] is that one of the hypotheses (1–4) must be given up. Here hypothesis (2) is given up, and a Dirac two-component wave function approach is used to describe the bound quark wave functions and spin. Hypothesis (5) is reinstated as isospin symmetry is assumed between the u and d quarks. Hypothesis (6a) is included, and the quark wave functions in the octet are taken to be spin dependent. Any gluons exchanged in the three quark system are assumed accounted for via a confining potential between the quarks.

We consider three quarks in the $(1/2^+)^3$ configuration. The $J = (3/2)\Delta$, $J =$ 1/2 nucleon mass difference shows the quark energies are spin dependent. It is natural to expect that the quark wave functions will also depend on spin. Spin-dependent forces can lead to different wave functions for quarks with spin parallel or antiparallel to the nucleon spin. In the Δ, or in any combination of the decuplet, a pair of quarks always interact in the $S = 1$ two quark spin state. In the neutron or proton or any of the octet systems, a quark pair interacts in a mixture of $S = 0$ or $S = 1$ two quark spin states. For a quark in the neutron, proton, or other octet member, the precise mixture of spin states depends on whether the quark z component of spin is parallel or antiparallel to the nucleon z component of spin. Consequently, with spin-dependent forces between quarks, the spin-up and spin-down wave functions can differ for quarks in the nucleon. This difference is because of spin-dependent forces, not z-dependent forces. The quark model wave function for the spin 1/2 proton, with spin up [5] is the three quark $(1/2^+)^3$ configuration:

$$\Psi = [2\mathrm{u}\uparrow \mathrm{u}\uparrow \mathrm{d}\downarrow_- \mathrm{u}\uparrow \mathrm{u}\downarrow \mathrm{d}\uparrow - \mathrm{u}\downarrow \mathrm{u}\uparrow \mathrm{d}\uparrow]\big/\sqrt{6} \tag{1}$$

with 2u quarks, each with charge of 2/3 the proton charge, and one d quark with charge minus 1/3 of the proton charge. The neutron spin-up wave function has just the u and d quarks interchanged in the above three quark wave function. Flavor (isospin) symmetry is assumed here between the u and d quark wave functions, but spin-dependent forces will be considered. The color part of the wave function is a totally antisymmetric determinant not shown. The wave function for other octet members has one or more strange quarks replacing the u or d quarks in the above wave function.

With the nucleon spin pointing up, two quarks have spin parallel to the nucleon spin z component, and one quark has spin antiparallel to the nucleon spin z component. A quark, with z component of spin parallel to the nucleon z component of spin, interacts with other quarks 75% in the $S = 1$ state, and 25% in the $S = 0$ spin state. A quark with z component of spin antiparallel to the nucleon z component of spin, interacts with the other quarks 50% in the $S = 1$ state, and 50% in the $S = 0$ state. Assuming spin-dependent quark–quark forces, the wave functions for the spin parallel and the spin antiparallel quarks can therefore differ. In the Delta, each quark interacts with other quarks always in the $S = 1$ state.

A fermion, with the proton charge e, bound in the $(1/2^+)$ state has the magnetic moment matrix element, in nuclear magnetons, of

$$\mu = -[4eM/3] \int F(r) r G(r)\, dr \tag{2}$$

Hence a bound quark, with charge q, has a magnetic moment of q/e times the above value. Magnetic moments of the bound quarks were reported [6] in the above units, with the 2/3 or $-1/3$ quark charges appearing as separate factors.

In terms of these matrix elements, with flavor symmetric forces but allowing for spin-dependent forces, the proton magnetic moment is

$$([2/3][4/3] + [1/3][2/3 - 1/3])\mu\uparrow + ([2/3][-1/3] + [1/3][2/3])\mu_{\mathrm{a}}\downarrow = \mu\uparrow . \tag{3}$$

The neutron magnetic moment is a different combination of these matrix elements:

$$\begin{aligned} &([2/3][-2/3] + [1/3][2/3 - 1/3])\mu\uparrow + ([2/3][2/3] + [1/3][-1/3])\mu_{\mathrm{a}}\downarrow \\ &= -\mu/3\uparrow + \mu_{\mathrm{a}}/3\downarrow . \end{aligned} \tag{4}$$

With the spin up and down radial wave functions the same, μ_{a} and μ would be the same, and then the neutron magnetic moment becomes $-2\mu/3\uparrow$, as a successful prediction of the quark model. To reproduce both the neutron and proton magnetic moments, the one quark magnetic moment matrix element must be larger (2.9466) for the z component of quark spin antiparallel to the nucleon spin z component, than for spin parallel case (2.793). Likewise, analysis of the octet magnetic moments [6] showed that the strange quark magnetic moment was 1.081 nm for the spin antiparallel case, and 1.753 for the spin parallel case.

The magnetic moment matrix element, μ, and the lower component contribution to the normalization, N_1, are related. If the lower component G, of the two component Dirac wave function, is r times the upper component F of the Dirac wave function, then the quark magnetic moment is proportional to N_1. The contribution to the normalization of these one quark wave functions for the upper component is,

$$N_0 = \int F^2(r)\, dr \tag{5}$$

For the lower component, the contribution to the normalization is,

$$N_1 = \int G^2(r)\, dr. \tag{6}$$

The notation denotes the orbital angular momentum $L = 0(1)$ for the upper (lower) component of the quark wave function, which in each component is coupled with spin to a total J of 1/2 for the $(1/2^+)$ wave function. Let N_{1a} be the lower component contribution to the norm for a quark with spin antiparallel to the nucleon.

Δu is the expectation value for u quarks with spin parallel to the nucleon spin minus the expectation value for u quarks with spin antiparallel to the nucleon spin. Δd is the same difference of expectation values only for d quarks in the nucleon. Neutrino experiment analyses [7–9] show the sum, $\Delta u + \Delta d$ to be 0.58 ± 0.01. The axial charge is the difference $\Delta u - \Delta d$, 1.2601. Using the $(1/2^+)^3$ octet wave function, Eq. (1), assuming isospin invariance between u and d quarks, but with spin-dependent forces, one finds,

$$\Delta u + \Delta d = 1 - [8N_1 - 4N_{1a}]/3 = 0.58 \tag{7}$$

and

$$|\mathrm{Ga/Gv}| = \Delta u - \Delta d = 5/3 - [16N_1 + 4N_{1a}]/9 = 1.2601 \tag{8}$$

To simultaneously reproduce [10] the proton axial charge, and the spin content of the nucleon data, the lower component contribution to the normalization of the one quark wave function, N_1, is 0.095 for spin parallel, and N_{1a} is 0.205 for spin antiparallel to the nucleon spin. This normalization difference can be well reproduced by the action of spin dependent forces. Neglecting any spin dependence to the wave functions, fitting the axial charge requires $N_1 = N_{1a} = 0.18$.

The purpose of this paper is to find a spin-dependent potential to be used with the Dirac equation, solutions from which will reproduce the above data. A previous solution to the Dirac equation with small quark masses [11] was based on a quadratic confining force, and it resulted in the lower component contribution to the norm being too large to fit the axial charge of the proton. Using a linear scalar confining potential [12, 13] only, the contribution of the lower component to the norm was too small. Using a linear confining and $1/r$ attractive potential, the axial charge could be fit, but the magnetic moment was too small [14]. So we consider here a $1/r$ attraction and a confining potential that has both linear and quadratic confining terms [15] and results in an analytic wave function. The purpose of using an analytic wave function is to obtain simple expressions for various matrix elements of interest.

2. THEORY

For the $(1/2^{+})$ state, the radial Dirac equation [7] for a one quark wave function is,

$$\begin{aligned} [m_{\mathrm{q}} + S - E + V]F + [-1/r - d/dr]G &= 0 \\ [-1/r + d/dr]F + [-m_{\mathrm{q}} - S - E + V]G &= 0 \end{aligned} \tag{9}$$

Here S and V are scalar and vector central potentials with the radial part of the upper and lower components of the wave function being F/r, and G/r respectively. The quark mass, m_{q} is neglected for the u and d quarks, but is about [1] 0.125 GeV for the strange quark, with values ranging from 0.080 to 0.155 GeV. To have the magnetic moment proportional to the lower component contribution to the normalization, a wave function is used where G is r times F. One such possible wave function is,

$$\begin{aligned} F &= Are^{-Lr}\exp[-1/2\alpha^2 r^2] \\ G &= Br^2 e^{-Lr}\exp[-1/2\alpha^2 r^2] \end{aligned} \tag{10}$$

These satisfy the Dirac equation if the scalar potential is,

$$S = [Z/6][Lr + \alpha^2 r^2] + 3L/2Zr \tag{11}$$

And the vector potential is,

$$V = [Z/6][Lr + \alpha^2 r^2] - 3L/2Zr, \tag{12}$$

where

$$\begin{aligned} Z &= \left[3\alpha^2 + m_{\mathrm{q}}^2\right]^{1/2} - m_{\mathrm{q}} \\ B/A &= -[E - m_{\mathrm{q}}]/3 \end{aligned} \tag{13}$$

And the energy is given from

$$E^2 = 3\alpha^2 + m_{\mathrm{q}}^2. \tag{14}$$

2.1. Nucleon Spin Dependent Potential Parameters

For the case of spin parallel to the nucleon spin, $L = 0.147$ GeV and $\alpha = 0.159$ GeV reproduce the quark magnetic moment and the lower component contribution to the norm of 0.205. This scalar and vector potential is mostly linear in the region where the wave function components are largest. For the spin antiparallel to the nucleon spin case, the potential parameters are $L = 0.172$ GeV and $\alpha = 0.070$ GeV. Taking the u, d quark masses to be zero, the one quark eigen-energies are 0.275 and 0.120 GeV respectively.

2.2. The Delta Particle

These potential fits can be used to estimate the potential acting on a quark in the Delta particle. Let x be the potential when two quarks interact in the spin 1 state, and y the potential when interacting in the spin zero state. Let V_a and S_a be the vector and scalar potentials of Eqs. (11, 12) fitted for the spin antiparallel case, and V, S for spin parallel case. Then

$$(x + y)/2 = V_\mathrm{a} + S_\mathrm{a}, \quad \text{and} \quad (3x + y)/4 = V + S, \tag{15}$$

and so

$$x = 2V - V_\mathrm{a} + 2S - S_\mathrm{a}. \tag{16}$$

Solving Eq. (14), for the potential acting in the Delta, one obtains the one quark eigen-energy as $E = 0.370$ GeV. Neglecting center of mass energy effects, one approximately expects the Delta energy as three times 0.370 or 1.110 GeV and the proton energy as two times $0.275 + 0.120$ or 0.670 GeV. These numbers differ from experiment (1.236 and 0.938) but tend to reproduce the experimental energy differences as from spin-dependent forces.

2.3. The Strange Quark

Using the potentials as parameterized for the u, d quarks, we solve for the strange quark magnetic moment for an octet member. The magnetic moments decrease with the mass assumed for the strange quark. The antiparallel spin case magnetic moment decreases with mass more rapidly than does the spin parallel case because of the spin-dependent potentials used. Using a strange quark mass [1] of 0.155 GeV, the magnetic moment is found to be 1.106 and 1.883 for the two spin states of a bound strange quark in the octet wave function. These values are close to the values (1.081 and 1.753) needed to empirically explain the octet magnetic moments.

3. CONCLUSIONS

We consider three quarks in the $(1/2^+)^3$ configuration. A Dirac equation approach is used with small current quark masses for the quarks. The $J = (3/2)\Delta$, $J = 1/2$ nucleon mass difference shows the quark energies are spin dependent.

It is natural to expect that the quark wave functions also depend on spin. Spin-dependent forces are found that lead to different wave functions for quarks with spin parallel or antiparallel to the nucleon spin. These potentials are fitted to the nucleon axial charge, magnetic moments, and spin content data. These potentials have a vector and a scalar part. The confining potential has both a linear and a quadratic term.

In the neutron or proton or any of the spin 1/2 octet systems, a quark pair interacts in a mixture of $S = 0$ or $S = 1$ two quark spin states. With the nucleon spin pointing up, two quarks have spin parallel to the nucleon spin z component, and one quark has spin antiparallel to the nucleon spin z component. A quark, with z component of spin parallel to the nucleon z component of spin, interacts with other quarks 75% in the $S = 1$ state, and 25% in the $S = 0$ spin state. A quark with z component of spin antiparallel to the nucleon z component of spin, interacts with the other quarks 50% in the $S = 1$ state, and 50% in the $S = 0$ state. In the Δ, or in any combination of the decuplet with total spin 3/2, a pair of quarks always interact in the $S = 1$ two quark spin state. Using the potentials fitted to reproduce the octet magnetic moment data, a potential is found for a quark in the Delta particle. Its eigen-energy is 0.370 GeV, compared to 0.275 and 0.120 GeV for quarks in the nucleon. These differing eigen-energies are about what is needed to explain the Delta nucleon mass difference.

Using the upper limit for the strange quark mass, this spin-dependent potential provides a reasonable fit to the bound strange quark magnetic moment as empirically determined from the octet magnetic moments.

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